Demography and Well-being

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Abstract

We present in this paper a deterministic model for a general human population dynamics. The main variables are the population by sex and the welfare variables. These variables are considered in the HDI[1], GDI[2] and GEM[3] calculation. The model has been validated for the case of Belgium in the period 1997-2008.

Nous présentons dans cet article un modèle déterministe pour une dynamique générale de la population humaine. Les principales variables sont la population par sexe et les variables du bien-être. Ces variables sont considérées dans le calcul de HDI, GDI et GEM. Le modèle a été validé pour le cas de la Belgique dans la période 1997-2008.

Key words: welfare variables, deterministic model, human population.
variables du bien-être, le modèle déterministe, la population humaine.

[1] Human Development Index, calculated by the UNO.
[2] Gender Development Index, calculated by the UNO.
[3] Gender Empowement Index, calculated by the UNO.

Introduction

The literature review shows that there are no models about the dynamics of human population considering both sexes, covering demographic processes such as births, deaths, migration, and depending on well-being. Therefore, we have created a deterministic mathematical model studying the evolution of human population per sexes, in which we have introduced three well-being variables.

These variables are being used by *UN* to determine the quality of life of a country. The variables are: the *Human Development Index*, (*HDI*), measures the average achievements in a country in three basic dimensions of human development: longevity, knowledge and a decent standard of living. The *Gender Empowerment Index*, (*GEM*), measures the level of opportunities for women. It is measured with three different dimensions: political participation and decision-marketing, economic participation and decision-marketing and control of economic resources. The *Gender Development Index (GDI)* is a social indicator similar to the *HDI*, which measures the social and economic inequalities between men and women.

The main property of this model is that is abstract, i.e., it is transferable to any country. This means that this model is a tool for the study of human populations and their quality of life.

The Model

First, all variables, whose have been used in this model, are defined in Appendix I alphabetically. Also, you can see the relation between these variables in the Forrester's Diagram (1961) (Figure I, Appendix III). This is the characteristic diagram of the System Dynamics.

The model reflects the influence of fertility, mortality, emigration, immigration on the male and female population, as well as the relation between fertility and mortality rates with well-being variables (*HDI, GEM, GDI*), turn these are calculated from input variables and the resulting population.

In Appendix II, you can observe the equations of the model which we show in this paper. The demographic equations will be explained in this section, where the demographic model is explained in detail.

Generic Equations

The starting point of this demographic model is presented by Micó et al. (2006):

$$\frac{dPOFL(t)}{dt}=\left(RFEF(t)\text{-}RDEF(t)+RINF(t)-REMF(t)\right)\cdot POFL(t) \quad (1)$$

$$\frac{dPOML(t)}{dt}=RFEM(t)\cdot POFL(t)+\left(RINM(t)-REMM(t)-RDEM(t)\right)\cdot POML(t) \quad (2)$$

In it, all variables depend on time $t\in[t_0,\infty[$.

There are male and female rates, so the subscript *i* represents for all variables both sex: *i = 1,* females and *i = 2,* males. Units of **fertility rates** (defined as female and male births among female population, respectively) for the three indices of quality of life are represented as outlined:

$$RFEF(t)=a_1(HDI(t),GDI(t),GEM(t)) \quad (3)$$

$$RFEM(t)=a_2(HDI(t),GDI(t),GEM(t)) \quad (4)$$

As for the dependence of **death rates** (defined as females and male deaths among female and male population, respectively) and indices are represented as:

$$RDEF(t)=b_1(HDI(t),GDI(t),GEM(t)) \quad (5)$$

$$RDEM(t)=b_2(HDI(t),GDI(t),GEM(t)) \quad (6)$$

Mathematical Structures

The structures (3), (4), (5) and (6) are described as:

$$RFEF(t) = \alpha_1 + \beta_1 \cdot HDI \cdot GDI \cdot GEM + \gamma_1 \cdot Cos(\mu_1 \cdot HDI \cdot GDI \cdot GEM) \quad (7)$$
$$RFEM(t) = \alpha_2 + \beta_2 \cdot HDI \cdot GDI \cdot GEM + \gamma_2 \cdot Cos(\mu_2 \cdot HDI \cdot GDI \cdot GEM) \quad (8)$$
$$RDEF(t) = \alpha_3 + \beta_3 \cdot HDI \cdot GDI \cdot GEM + \gamma_3 \cdot Cos(\mu_3 \cdot HDI \cdot GDI \cdot GEM) \quad (9)$$
$$RDEM(t) = \alpha_4 + \beta_4 \cdot HDI \cdot GDI \cdot GEM + \gamma_4 \cdot Cos(\mu_4 \cdot HDI \cdot GDI \cdot GEM) \quad (10)$$

In this paper we wanted to get generic formulas for deaths and fertility rates by sex. The relations have been searched between rates and each quality of life index, and we have chosen some counties, OECD countries, to observe the relation. These dates show a common tendency to complete cycles. The real data available are for a short period of time, which prevents suit a sum of logistics, as discussed in Marcheti et al. (1996).

The structures (7), (8), (9) and (10) are the result of the dependence of each variable separately, also this is the best option to fit a model of cycles with few real data.

The fertility and mortality rates depend on welfare variables, *HDI*, *GDI* and *GEM,* and these depend on other input variables. These calculations are explained in reports of the United Nations Development (*UNDP*, 1990-2008). And all input variables with a time dependence (necessary for the calculation of the quality of life indices, you can see it in Appendix I) have been fitted as sums of logistics functions.

On the other hand, the **emigration and immigration rates** have been fitted as logistic functions with positive parameters, Micó et al., (2006):

$$REMF(t) = \eta_5 + \frac{\beta_5}{1+\gamma_5 \cdot exp\left(\alpha_5 \cdot (t-\mu_5)\right)} + \frac{\varphi_5}{1+\rho_5 \cdot exp\left(\omega_5 \cdot (t-\tau_5)\right)} \quad (11)$$
$$REMM(t) = \eta_6 + \frac{\beta_6}{1+\gamma_6 \cdot exp\left(\alpha_6 \cdot (t-\mu_6)\right)} + \frac{\varphi_6}{1+\rho_6 \cdot exp\left(\omega_6 \cdot (t-\tau_6)\right)} \quad (12)$$
$$RINF(t) = \eta_7 + \frac{\beta_7}{1+\gamma_7 \cdot exp\left(\alpha_7 \cdot (t-\mu_7)\right)} + \frac{\varphi_7}{1+\rho_7 \cdot exp\left(\omega_7 \cdot (t-\tau_7)\right)} \quad (13)$$
$$RINM(t) = \eta_8 + \frac{\beta_8}{1+\gamma_8 \cdot exp\left(\alpha_8 \cdot (t-\mu_8)\right)} + \frac{\varphi_8}{1+\rho_8 \cdot exp\left(\omega_8 \cdot (t-\tau_8)\right)} \quad (14)$$

The mathematical structures of the functions 11, 12, 13 and 14 are appointed by Marchetti et al. (1996). These have been calculated respect to time, as in papers of Marchetti, et al., (1996), Micó and Caselles (1998), and Chowdhury and Allen (2001).

The independent variables of these rates (*REMM, REMF, RINM, RINF*) are the time. They don't depend on well-being indices, because if you consider this case, you'll define a world divided by areas, and these rates will depend on the quality of life of the input country and the quality of life of the output country.

All rates mathematical structures have been found with a process of trial and error and using a mathematical program, Regint, (Caselles, 2008).

The real data has been extracted from Eurostat. In the case of fertility and death rates for the period 1994-2007, and emigration and immigration rates for the period 1997-2007. The historical data of well-being indices, *GDI*, *HDI* and *GEM*, has been extracted from reports of the United Nations Development (*UNDP*), for the period 1993-2007.

All functions have been validated because the residuals are random, the coefficients of the determination are high and acceptance of the normality of the data to meet the Kolmogorov-Smirnov test. You can see it in Appendix IV, figures 2 to 9.

Model validation

For **deterministic validation** the model has been written as a set of finite difference equations and their solutions have been calculated with the Euler approximation. This method was chosen because

some documents, such as Djidjeli et al. (1998), explained that the Euler method is the best suited to solve such equations. Also Djidjeli et al. (1998) make a comparison with the model of Runge-Kutta and say that the latter often gives false results. On the other hand, Letellier et al. (2004) say that the Euler method is often used with different models, and with only a small discretization time, these solutions are equivalent to the continuous model.
The software tool used for the validation of the model has been the SIGEM. The validation has been realized for the period 1997-2008.
The validation process has been considered successful for three reasons:

- The graphics overlay of historical and calculated data is good.
- The determination coefficients, R^2, are very high. R^2 is a useful index for the overall fit:

$$R^2 = \frac{\left(\sum_i (x_i-\mu_x)(y_i-\mu_y)\right)^2}{\sum_i (x_i-\mu_x)^2 \sum_i (y_i-\mu_y)^2} \qquad (15)$$

where (x_i, y_i) is the data being compared and μ_x, μ_y are the average values of these data, respectively.

- The randomness of residuals has been verified by the maximum relative error.

To verify this information, sees Figures 10, 11 and 12 in Appendix V. The deterministic validation may be considered successful as all the R^2 were above 0.99 and the relative errors did not exceed 5%.

Conclusion

An abstract model is presented to study the human population dynamics with differentiation per gender, in which one of the well-being variables presented by the United Nations is included: the *HDI, GDI* and *GEM*.
One model is presented to undertake a more detailed and complete study: the deterministic model. Fertility, death, emigration and immigration rates, per gender, were validated with the deterministic model for the specific case of Belgium in the period 1997-2008, and the corresponding fitted functions obtained higher determination coefficient values, R^2, than those obtained in previous studies (Sanz et al. 2009).
One approach could be obtained by the conversion of this deterministic model into a stochastic model
Other future research works could consist of attempts to validate this model in other world countries where data are available. In these works, the first task will entail verifying that the structures of the formulae obtained herein for fertility, death and migration rates are actually generic as they are valid for all these countries.
Finally another possible, although ambitious, work could consist in attempting to find a model that considers several countries or regions, in which the migration rates could be fitted in terms of the well-being variables corresponding to the countries involved.

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APPENDIX I

List of variables involved in the model.

Demography:
DEFE Female Deaths [population] (flow variable); **DEMA** Male Deaths [population] (flow variable); **DETO** Total Deaths [population] (auxiliary variable); **EMIF** Female Emigration [population] (flow variable); **EMIG** Total Emigration [population] (auxiliary variable); **EMIM** Male Emigration [population] (flow variable); **POFI** Female Population at the beginning of the year [population] (constant); **POFL** Female Population at the end of the year [population] (level variable); **POMI** Male Population at the beginning of the year [population](constant); **POML** Male Population at the end of the year [population] (level variable); **POPI** Population at the beginning of the year [population] (auxiliary variable): **POPL** Population at the end of the year [population] (auxiliary variable); **PRPF** Females proportion (auxiliary variable); **PRPM** Males proportion (auxiliary variable); **RDEF** Females Death Rate [%] (auxiliary variable); **RDEM** Males Death Rate [%] (auxiliary variable); **REMF** Female emigration rate [%] (auxiliary variable); **REMM** Male emigration rate [%] (auxiliary variable); **RFEF** Female Fertility rate [births/female population] (auxiliary variable); **RFEM** Male Fertility rate [births/female population] (auxiliary variable); **RINF** Female immigration rate [%] (auxiliary variable); **RINM** Male immigration rate [%] (auxiliary variable); **TEMI** Initial year [time] (constant); **TEMS** Year [time] (level variable); **XACF** Female Births [population] (flow variable); **XACI** Total Births [population] (auxiliary variable); **XACM** Male Births [population] (flow variable); **YNMF** Female immigration [population] (flow variable); **YNMI** Total immigration [population] (auxiliary variable); **YNMM** Male immigration [population] (flow variable)

Human Development Index:
GDPR Gross Domestic Product [PPP US$] (setting variable); **GRRR** Gross Rate Registered to level primary secondary and tertiary [%] (auxiliary variable); **LAPO** Literate Adult Population [%] (auxiliary variable); **LEBI** Life expectancy at birth [age] (auxiliary variable); **XHDI** Human development Index [%] (level variable); **YEDU** Educational Index [%] (auxiliary variable); **YGDP** Gross Domestic Product Index [%] (auxiliary variable); **YGRR** Gross Rate Registered to level primary secondary and tertiary index[%] (auxiliary variable); **YLAP** Literacy Rate Adults [%] (auxiliary variable) **YLEB** Life expectancy at birth index [%] (auxiliary variable)

Gender Development Index:
GRFE Female Gross Registered to level primary, secondary and tertiary [%] (setting variable); **GRMA** Male Gross Registered to level primary, secondary and tertiary [%] (setting variable); **LEBF** Female life expectancy at birth [age] (setting variable); **LEBM** Male life expectancy at birth [age] (setting variable); **RLAF** Female literacy rate adults [%] (auxiliary variable); **RLAM** Male literacy rate adults [%] (auxiliary variable); **RLIF** Female literacy rate [%] (setting variable); **RLIM** Male literacy rate [%] (setting variable); **XGDI** Gender Development Index [%] (level variable); **YEID** Equally Distributed Education Index [%] (auxiliary variable); **YEFE** Female Education Index [%] (auxiliary variable); **YEMA** Male Education Index [%] (auxiliary variable); **YEVD** Equally Distributed life expectancy at birth index [%] (auxiliary variable); **YFEM** Female income [PPP US$] (setting variable); **YGRF** Female Gross Rate Registered to level primary secondary and tertiary [%] (auxiliary variable); **YGRM** Male Gross Rate Registered to level primary secondary and tertiary [%] (auxiliary variable); **YIID** Equally Distributed Income Index [%] (auxiliary variable); **YIFE** Female income index [%] (auxiliary variable); **YIMA** Male income index [%] (auxiliary variable);

YLEF Female life expectancy at birth index [%] (auxiliary variable); **YLEM** Male life expectancy at birth index [%] (auxiliary variable); **YMAL** Male income [PPP US$] (setting variable)

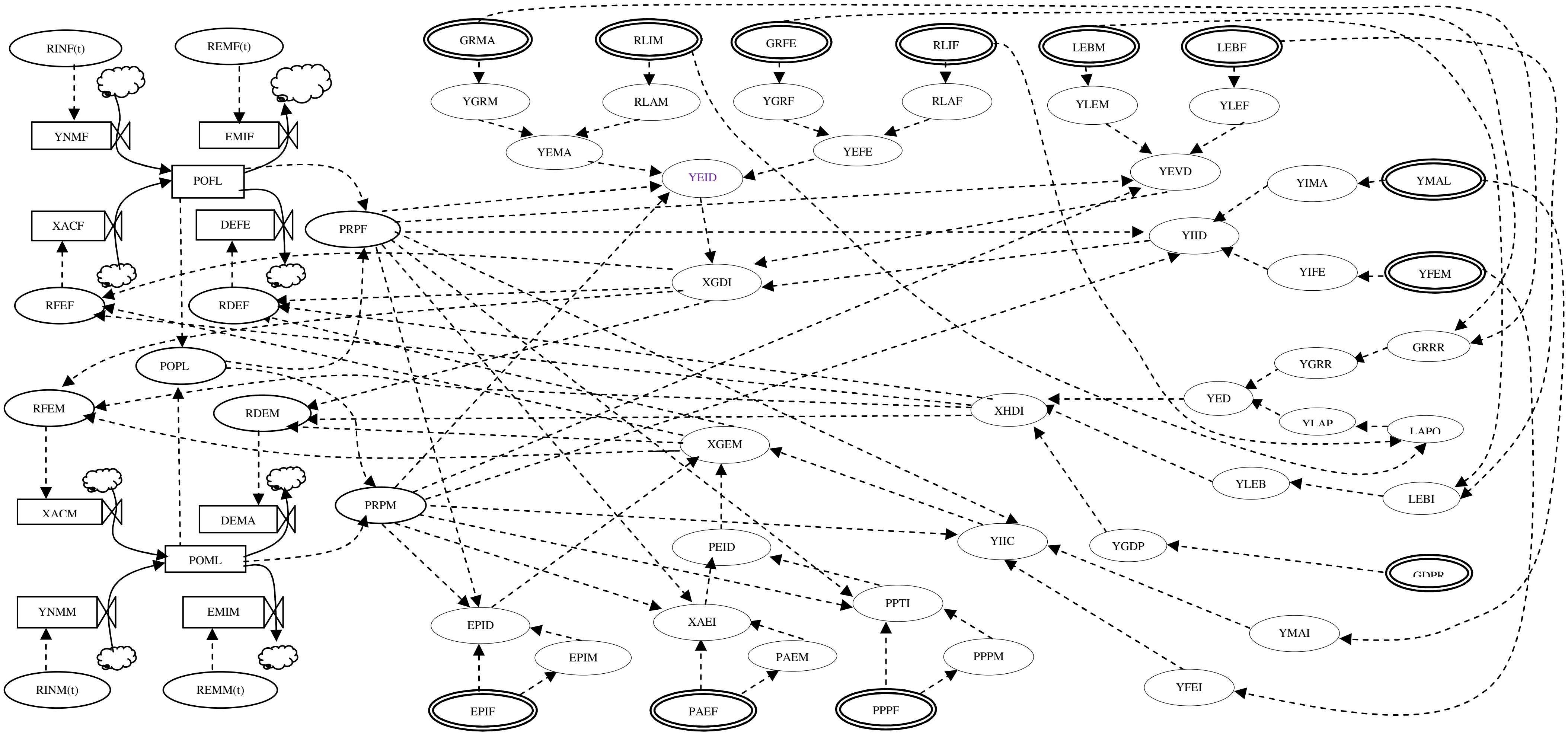
Gender Empowerment Index:

EPID Percentage parliamentary representation [%] (auxiliary variable); **EPIF** Female Percentage parliamentary representation [%] (setting variable); **EPIM** Male Percentage parliamentary representation [%] (auxiliary variable) ; **PAEF** Female percentage shares of positions as legislators senior officials and managers [%] (setting variable); **PAEM** Male percentage shares of positions as legislators senior officials and managers [%] (auxiliary variable); **PEID** Average PEID [%] (auxiliary variable); **PPPF** Female percentage shares of professional and technical positions [%](setting variable); **PPPM** Male percentage shares of professional and technical positions [%](auxiliary variable); **PPTE** PEID [%] (auxiliary variable); **PPTI** Professional index [%] (auxiliary variable); **XAEE** PEID within index [%] (auxiliary variable); **XAEI** PEID index senior [%] (auxiliary variable); **XGEM** Gender Empowerment Index [%] (level variable); **YFEI** Female income [PPP US$] (auxiliary variable); **YIIC** Total income [PPP US$] (auxiliary variable); **YMAI** Male Income [PPP US$] (auxiliary variable)

APPENDIX II

RFEF rfef=(402.194)+(-0.0209906)*xidh*xidg*xipg+(-402.17)*Cos(0.00909603*xidh*xidg*xipg)
RFEM rfem=(515.211)+(-0.025085)*xidh*xidg*xipg+(-515.19)*Cos(0.00879168*xidh*xidg*xipg)
RDEF rdef=(8.58545)+(-12.8429)*xidh*xidg*xipg+(-4.89842)*Cos(2.66067*xidh*xidg*xipg)
RDEM rdem=(-3082)+(0.110787)*xidh*xidg*xipg+(3082.08)*Cos(0.00838199*xidh*xidg*xipg)
REMF remf=(0.00538) + (0.000723221)/(1 + 1.76775*Exp(2.1239*(1997 - tems))) + (0.000801337)/(1 + 10.648*Exp(1.50677*(2002 - tems)))
REMM remm=(0.0078099) + (0.000729677)/(1 + 9.33369*Exp(2.41473*(1997 - tems))) + (0.00169915)/(1 + 8.12132*Exp(1.18132*(2002 - tems)))
RINF rinf=(263.108) + (-263.095)/(1 + (-0.000026637)*Exp(0.112474*(1997 - tems))) + (0.000823442)/(1 + 390975*Exp(3.65354*(2001 - tems)))
RINM rinm=(0.00831431) + (0.00394115)/(1 + 19.1101*Exp(1.08366*(1997 - tems))) + (0.00317799)/(1 + 57.7691*Exp(1.96601*(2003 - tems)))
TEMS tems=temi+dt
LEBI lebi=(lebf+lebm)/2
YLEB yleb=(lebi-25)/(85-25)
YGDPygdp=(Log(gdpr)-Log(100))/(Log(40000)-Log(100))
GRRR grrr=(grfe+grma)/2
YGRR ygrr=(grrr-0)/(100-0)
LAPO lapo=(rlif+rlim)/2
YLAP ylap=(lapo-0)/(100-0)
YEDU yedu=(2/3)*ylap+(1/3)*ygrr
XHDI xhdi=(1/3)*(yleb+ygdp+yedu)
YLEF ylef=(lebf-27.5)/(87.5-27.5)
YLEM ylem=(lebm-22.5)/(82.5-22.5)
YEVD yevd=1/((prpf/ylef)+(prpm/ylem))
YGRF ygrf=((grfe)-0)/(100-0)
RLAF rlaf=(rlif-0)/(100-0)
YEFE yefe=((2/3)*rlaf)+((1/3)*ygrf)
YGRM ygrm=((grma)-0)/(100-0)
RLAM rlam=(rlim-0)/(100-0)
YEMA yema=((2/3)*rlam)+((1/3)*ygrm)
YEID yeid=1/((prpf/yefe)+(prpm/yema))
YIMA yima=(Log(ymal)-Log(100))/(Log(40000)-Log(100))
YIFE yife=(log(yfem)-log(100))/(Log(40000)-log(100))
YIID yiid=1/((prpf/yife)+(prpm/yima))
XGDI xgdi=(1/3)*(yevd+yeid+yiid)
EPIM epim=100-epif
EPID epid=(1/((prpf/epif)+(prpm/epim)))/50
PPPM pppm=100-pppf
XAEE xaee=1/((prpf/pppf)+(prpm/pppm))
XAEI xaei=xaee/50
PAEM paem=100-paef
PPTE ppte=1/((prpf/paef)+(prpm/paem))
PPTI ppti=ppte/50
PEID peid=(xaei+ppti)/2
YMAI ymai=(ymal-100)/(40000-100)
YFEI yfei=(yfem-100)/(40000-100)
YIIC yiic=1/((prpf/yfei)+(prpm/ymai))
XGEM xgem=(1/3)*(epid+peid+yiic)
POPI popi=pofi+pomi
PRPF prpf=pofi/popi
PRPM prpm=pomi/popi
XACF xacf=pofi*rfef
DEFE defe=pofi*rdef/100
YNMF ynmf=pofi*rinf
EMIF emif=pofi*remf
POFL pofl=pofi+xacf+ynmf-emif-defe
XACM xacm=pomi*rfem
DEMA dema=pomi*rdem/10
YNMM ynmm=pomi*rinm
EMIM emim=pomi*remm
POML poml=pomi+xacm+ynmm-emim-dema
XACI xaci=xacf+xacm
DETO deto=defe+dema
YNMI ynmi=ynmf+ynmm
EMIG emig=emif+emim
POPL popl=pofl+poml

APPENDIX III



APPENDIX IV

Rates. Parameters.

RFEF i=1	α_i	402.194
	β_i	-0.0209906
	μ_i	0.00909603
	γ_i	-402.17

Table 1: Parameters' values for $RFEF(hdi, gdi, gem)$

RFEM i=2	α_i	515.211
	β_i	-0.025085
	μ_i	0.00879168
	γ_i	-515.19

Table 2: Parameters' values for $RFEM(hdi, gdi, gem)$

RDEF i=3	α_i	8.58545
	β_i	-12.8429
	μ_i	2.66067
	γ_i	-4.89842

Table 3: Parameters' values for $RDEF(hdi, gdi, gem)$

RDEM i=4	α_i	-3082
	β_i	0.110787
	μ_i	0.00838199
	γ_i	3082.08

Table 4: Parameters' values for $RDEM(hdi, gdi, gem)$

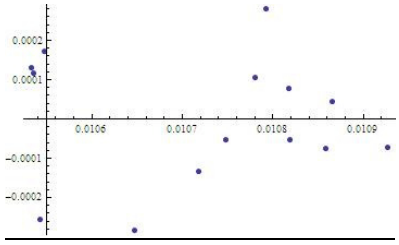


Figure 2: Residues (ordinate) of Female Birth Rate (abscise) in Belgium, from the welfare variables in the period 1993-2007. R^2=0.422147. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha, 15) > 0.129012$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

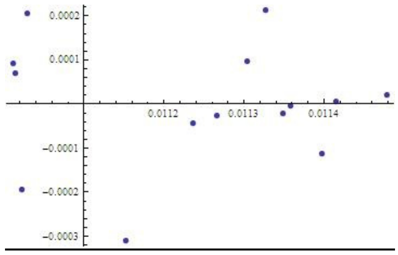


Figure 3: Residues (ordinate) of Male Birth Rate (abscise) in Belgium, from the welfare variables in the period 1993-2007. R^2 =0.57464. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha, 15) > 0.100662$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

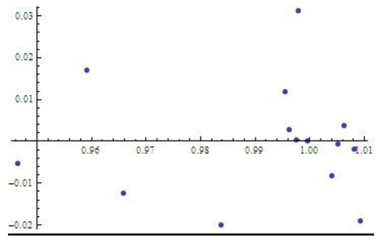


Figure 4: Residues (ordinate) of Female Death Rate (abscise) in Belgium, from the welfare variables in the period 1993-2007. R^2 = 0.677924. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha, 15) > 0.176274$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

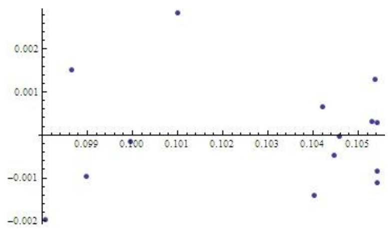


Figure 5: Residues (ordinate) of Male Death Rate (abscise) in Belgium, from the welfare variables in the period 1993-2007. R^2 = 0.834443. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha, 15) > 0.118085$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

REMF i=5	η_i	0.00538
	β_i	0.000723221
	μ_i	1997
	γ_i	2.1239
	φ_i	0.000801337
	ρ_i	10.648
	ω_i	1.50677
	τ_i	2002

Table 5: Parameters' values for *REMF*(t)

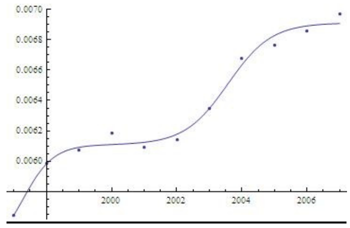


Figure 6: Fitted function (solid line) and real data (dots) for Female Emigration Rate along time in Belgium, in the period 1997-2007. R^2 = 0.988992. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha,11) > 0.212295$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

REMM i=6	η_i	0.0078099
	β_i	0.000729677
	μ_i	1997
	γ_i	9.33369
	φ_i	0.00169915
	ρ_i	8.12132
	ω_i	1.18132
	τ_i	2002

Table 6: Parameters' values for *REMM*(t)

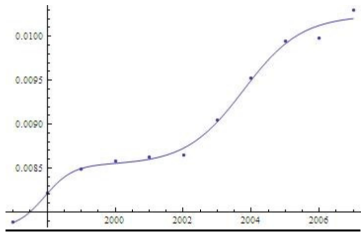


Figure 7: Fitted function (solid line) and real data (dots) for Male Emigration Rate along time in Belgium, in the period 1997-2007. R^2 = 0.993854. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha,11) > 0.222535$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

RINF i=7	η_i	263.108
	β_i	-263.095
	μ_i	1997
	γ_i	-0.000026637
	φ_i	0.000823442
	ρ_i	390975
	ω_i	3.65354
	τ_i	2001

Table 7: Parameters' values for *RINF*(t)

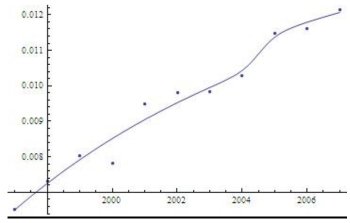


Figure 8: Fitted function (solid line) and real data (dots) for Female Immigration Rate along time in Belgium, in the period 1997-2007. R^2 = 0.975102. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha,11) >$ 0.167186, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the

RINM i=8	η_i	0.00831431
	β_i	0.00394115
	μ_i	1997
	γ_i	19.1101
	φ_i	0.00317799
	ρ_i	57.7691
	ω_i	1.96601
	τ_i	2003

Table 8: Parameters' values for *RINM*(t)

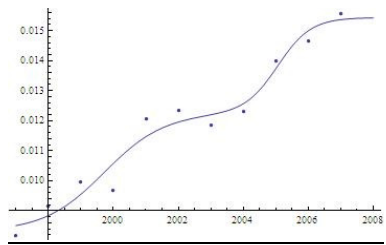


Figure 9: Fitted function (solid line) and real data (dots) for Male Immigration Rate along time in Belgium, in the period 1997-2007. R^2 = 0.961916. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha,11) > 0.156849$, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

APPENDIX V

Deterministic Validation for the period 1997-2008.

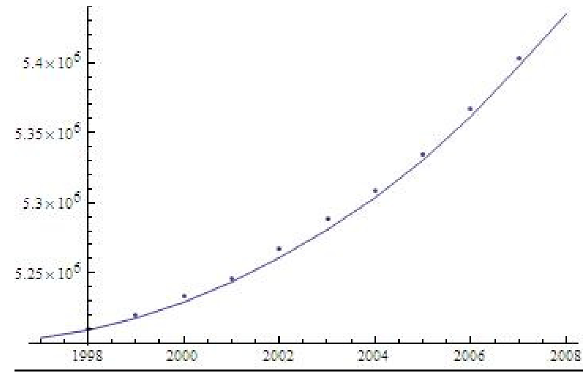


Figure 10: Forecast function (solid line) given by the model and real data (dots) for Belgium's Female Population, in the period 1997-2008, R^2 = 0.99608, with 0.140559% of maximum relative error. The model is considered validated, since the error does not exceed 5%.

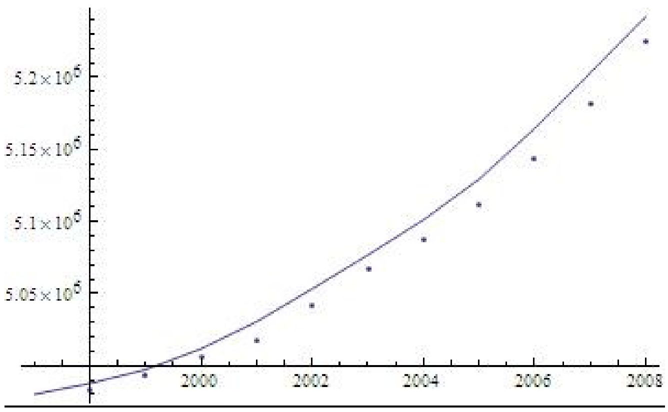


Figure 11: Forecast function (solid line) given by the model and real data (dots) for Belgium's Male Population, in the period 1998-2005, R^2 = 0.975076, with 0.418226 % of maximum relative error. The model is considered validated, since the error does not exceed 5%.

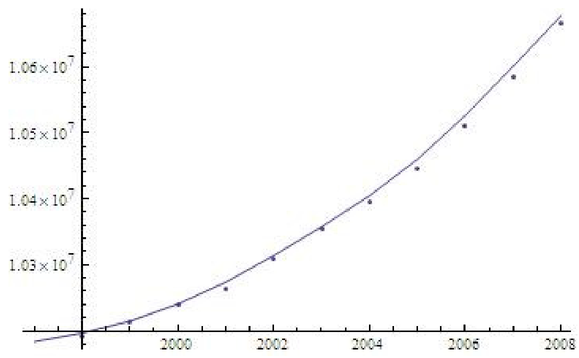


Figure 12: Forecast function (solid line) given by the model and real data (dots) for Belgium's Population, in the period 1998-2005, R^2 =0.996335 with 0.151032% of maximum relative error. The model is considered validated, since the error does not exceed 5%.